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Vector and axial-vector mesons in Quasilocal Quark Models

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Abstract. We consider the $SU(2)$ Quasilocal Quark Model of the NJL-type including vector and axial-vector four-fermion interaction vertices with derivatives. The mass spectrum and a set of model-independent relations for the ground and first-excited states are calculated. The chiral-symmetry restoration sum rules in these channels are imposed for matching to QCD at intermediate energies in order to get a number of constraints on parameters of the $SU(2)$ QQM.

PACS. 12.40.Yx Hadron mass models and calculations

1 Introduction and motivation

One of the important problems of hadron physics is the description of low-energy spectral characteristics for light vector (V) and axial-vector (A) mesons as this sector is related to a number of physical observables. In particular, recent experimental data of ALEPH [1] and OPAL [2] collaborations on hadronic τ-decays indicate that, in order to check both perturbative and non-perturbative QCD features, it is necessary to take into account more degrees of freedom in vector channels for the VV-AA correlation functions. Namely, in [3] an analysis has been performed of experimental data obtained by the ALEPH collaboration for the correlation function difference $\Pi^{\mathrm{V}} - \Pi^{\mathrm{A}}$ at intermediate energies of ≤ 3 GeV. A small contribution of the first radial excitations of vector mesons to $\Pi^{\mathrm{V}} - \Pi^{\mathrm{A}}$ can be seen experimentally, but the contribution of the next ones is nearly negligible. Moreover, as is known from the experimental data [4] and the theoretical investigations [5,6] there exists a series of heavier meson states, $\rho(1450)$, $\rho(2150)$, ... whose quantum numbers are those of the $\rho(770)$-meson and which represent radial excitations of the $\rho(770)$-meson in terms of potential quark models. In the axial-vector channel the excitations of the $a_1(1260)$ resonance such as $a_1(1640)$ can exist; however, its mass spectrum has not been yet accurately identified [4–7].

In order to describe the spectral characteristics of VA-mesons and a set of the low-energy coupling constants taking into account excited states, the Quasilocal Quark Models (QQM) were introduced in [8]. The QCD-motivated QQM are sufficiently general and allow relatively easily to get a wide set of the spectral-mass relations for the vector and axial-vector mesons and their excitations. Based on the quasilocal quark interactions realized by higher-dimensional quark operators with derivatives and on the Dynamical Chiral-Symmetry Breaking (DCSB), additional meson states are created for sufficiently strong-coupling constants.

Such a quasilocal approach (see also [9]) represents a systematic extension of the NJL-model [10] towards the complete effective action of QCD where many-fermion vertices with derivatives possess the manifest chiral symmetry of interaction, motivated by the soft momentum expansion of the perturbative QCD effective action. In the effective action of the Quasilocal Quark Models of the NJL type the low-energy gluon effects are hidden in the coupling constants. Alternative schemes including the condensates of low-energy gluons can be found in [11].

At the same time in the large-N_c approach, which is equivalent to planar QCD [12], the correlators for color-singlet quark currents are saturated by narrow meson resonances. On the other hand, their high-energy asymptotics is provided [13] by the perturbation theory and the Operator Product Expansion (OPE) due to asymptotic freedom of QCD. The differences of opposite-parity correlators, which are zero in the chiral limit of perturbation theory, reveal a fast decrease at large Euclidean momentum. Thus, the chiral symmetry is restored at high energies. Comparison of these two approaches allows to obtain a set of Chiral-Symmetry Restoration (CSR) sum rules. Since the QQM deals with a few low-lying resonances, the correlators of the QQM can be matched [14] at intermediate energies to the OPE of QCD correlators. This matching realizes the correspondence to QCD and improves the predictability of the QQM at intermediate energies.

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In the present work the vector-axial-vector version of $SU(2)$ QQM is considered with two channels, where two pairs of vector and axial-vector mesons are generated. Respectively, it is expected to reproduce the lower part of the QCD VA-meson spectrum in the planar limit and the leading asymptotics of chiral-symmetry restoration for higher energies. In sect. 2 we define the VA $SU(2)$ QQM with two pairs of VA-mesons and the corresponding mass spectrum for VA boson states is obtained. With the help of the four-resonance ansatz for VA-mesons the correlators of the VA, $SU(2)$ are matched to the OPE of QCD correlators and a number of constraints on parameters of the QQM from CSR sum rules are performed in sect. 3. We summarize our results and conclusions concerning the further development in sect. 4.

2 SU(2) Quasilocal Quark Model for the vector and axial-vector mesons

The $SU(2)$ QQM Lagrangian for the two-channel vector and axial-vector case in the chiral limit $m_q = 0$ has the form [15] (in Euclidean space)

$$\mathcal{L}_{\mathrm{VA}} = \bar{q} i \not{\partial} q + \frac{1}{4N_f N_c \Lambda^2} \times \sum_{k,l=1}^{2} b_{kl} \left[\bar{q}\Gamma^i_{\mathrm{V},k} q \cdot \bar{q}\Gamma^i_{\mathrm{V},l} q + \bar{q}\Gamma^i_{\mathrm{A},k} q \cdot \bar{q}\Gamma^i_{\mathrm{A},l} q \right], \quad (1)$$

$$\Gamma^i_{\mathrm{V},k} \equiv i\gamma_\mu f_k(s)\tau^i, \quad \Gamma^i_{\mathrm{A},k} \equiv i\gamma_\mu\gamma_5 f_k(s)\tau^i, \quad i = 1,2,3,$$

where $q \equiv q_j$ (j is the number of flavors N_f) are color fermionic fields with N_c components, b_{kl} represents the symmetric non-singular matrix of real coupling constants, and τ^i denote Pauli matrices. The quantities $f_k(s)$, $s \to -\partial^2/\Lambda^2$ are the form factors specifying the quasilocal interaction. We accept the following sequence of action of the derivatives for the Hermitian fermion currents:

$$\bar{q}\frac{\partial^2}{\Lambda^2} q = \frac{1}{4}\bar{q}\left(\frac{\overrightarrow{\partial} - \overleftarrow{\partial}}{\Lambda}\right)^2 q. \quad (2)$$

In addition, let us regularize the interaction vertices by introducing the momentum cutoff

$$\bar{q}q \longrightarrow \bar{q}\theta(\Lambda^2 + \partial^2) q, \quad (3)$$

and choose the polynomial form factors as being orthogonal on the unit interval,

$$\int_0^1 f_k(s) f_l(s) \mathrm{d}s = \delta_{kl}. \quad (4)$$

We select out here

$$f_1(s) = 2 - 3s, \qquad f_2(s) = -\sqrt{3}s. \quad (5)$$

As this model interpolates the low-energy QCD action, it is supplied with a cutoff Λ (of order of the CSB scale) for virtual quark momenta in quark loops. It is convenient to pass to the auxiliary vector (ρ^i_μ) and axial-vector (a^i_μ) fields,

$$\mathcal{L}_{\mathrm{aux}} = \bar{q} i\not{\partial} q + \sum_{k=1}^{2} i\bar{q}\left(\Gamma^i_{\mathrm{V},k}\rho^i_{k,\mu} + \Gamma^i_{\mathrm{A},k} a^i_{k,\mu}\right) q + N_f N_c \Lambda^2 \sum_{k,l=1}^{2} \left(\rho^i_{k,\mu} b^{-1}_{kl} \rho^i_{l,\mu} + a^i_{k,\mu} b^{-1}_{kl} a^i_{l,\mu}\right). \quad (6)$$

After integrating out the quark fields

$$\left\langle \exp\left(-\int \mathrm{d}^4 x\, \mathcal{L}\right)\right\rangle_{\bar{q}q} \equiv \exp(-S_{\mathrm{eff}}),$$

one comes to the bosonic effective action

$$S_{\mathrm{eff}}(\rho^i_{\mu,k}, a^i_{\mu,k}) = N_f N_c \Lambda^2 \times \sum_{k,l=1}^{2} \{\rho^i_{k,\mu} b^{-1}_{kl} \rho^i_{l,\mu} + a^i_{k,\mu} b^{-1}_{kl} a^i_{l,\mu}\} - N_f N_c \mathrm{Tr} \ln \not{D}|_{\mathrm{reg}},$$

$$\not{D} \equiv i(\not{\partial} + M) + i\sum_{k=1}^{2}\left(\Gamma^i_{\mathrm{V},k}\rho^i_{k,\mu} + \Gamma^i_{\mathrm{A},k} a^i_{k,\mu}\right), \quad (7)$$

where we have introduced the dynamic mass function $M \equiv \sum_k \sigma_k f_k(s)$, with σ_k being the vacuum expectation values of scalar fields [16]. We use the chirally invariant regularization of the fermionic determinant

$$\ln\det\not{D} = \mathrm{Tr}^{\mathrm{all}} \ln\not{D} \longrightarrow N_f N_c \mathrm{Tr}\ln\not{D}|_{\mathrm{reg}} \equiv \frac{1}{2} N_f N_c \mathrm{Tr}\ln\frac{\not{D}\not{D}^\dagger}{\mu^2},$$

where the constant μ is a normalization scale for quark fields and the trace "Tr" is assumed over all degrees of freedom except the color and flavor ones. We will carry out a further analysis in the mean-field approximation ($N_c \gg 1$). Expanding eq. (7) in boson fields and retaining the quadratic in fields part only, one obtains

$$S^{(2)}_{\mathrm{eff}}(\rho^i_{\mu,k}, a^i_{\mu,k}) = \frac{1}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \times \sum_{k,l=1}^{2} [\rho^i_{k,\mu} C^{\rho,\mu\nu}_{kl} \rho^i_{l,\nu} + a^i_{k,\mu} C^{a,\mu\nu}_{kl} a^i_{l,\nu}]. \quad (8)$$

The inverse propagators are defined by the corresponding second variation of $S_{\mathrm{eff}}(\rho^i_{\mu,k}, a^i_{\mu,k})$:

$$C^{(\rho,a)\mu\nu}_{kl} = 2N_f N_c \Lambda^2 b^{-1}_{kl}\delta_{\mu\nu} - N_f N_c \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \times \mathrm{tr}\left\{ (i\gamma_\mu, i\gamma_\mu\gamma_5) \frac{f_k\left(\left(\frac{q+p/2}{\Lambda}\right)^2\right)}{\not{q} + \frac{1}{2}\not{p} + iM} \times (i\gamma_\nu, i\gamma_\nu\gamma_5) \frac{f_l\left(\left(\frac{q-p/2}{\Lambda}\right)^2\right)}{\not{q} - \frac{1}{2}\not{p} + iM} \right\}, \quad (9)$$

where the trace "tr" spans the Dirac indices only.

To obtain mass spectrum we expand expression (9) in a small external momentum p ($p^2/\Lambda^2 \ll 1$) up to terms $\sim p^2$ and calculate the corresponding loop integral using the momentum cutoff regularization. To compensate the quadratic divergences in this integral, we parametrize the matrix of coupling constants as follows:

$$16\pi^2 b_{kl}^{-1} = \delta_{kl} - \frac{4}{3}\frac{\bar{\Delta}_{kl}}{\Lambda^2}; \qquad \bar{\Delta}_{kl} \ll \Lambda^2. \tag{10}$$

The general structure of (9) is

$$C_{kl}^{(\rho,a)\mu\nu} = \frac{N_f N_c}{12\pi^2}\left[\left(\hat{A}_{kl}^{(\rho,a)}p^2 + \hat{B}_{kl}^{(\rho,a)}\right)\delta_{\mu\nu} - \hat{A}_{kl}^{(\rho,a)}p_\mu p_\nu\right] + O\left(\frac{M_0^2}{\Lambda^2}\right), \tag{11}$$

where the two symmetric matrices —the kinetic term $\hat{A}$ and the momentum-independent part $\hat{B}$— have been introduced:

$$\hat{A}^{(\rho,a)} \equiv \begin{pmatrix} 4\ln\frac{\Lambda^2}{M_0^2} - \frac{15}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{3}{2} \end{pmatrix}, \tag{12}$$

$$\hat{B}^{\rho} \equiv \begin{pmatrix} -2\bar{\Delta}_{11} & -2\bar{\Delta}_{12} \\ -2\bar{\Delta}_{12} & -2\bar{\Delta}_{22} \end{pmatrix}, \tag{13}$$

$$\hat{B}^{a} \equiv \begin{pmatrix} -2\bar{\Delta}_{11} + \sigma_{11} & -2\bar{\Delta}_{12} + \sigma_{12} \\ -2\bar{\Delta}_{12} + \sigma_{12} & -2\bar{\Delta}_{22} + \sigma_{22} \end{pmatrix}, \tag{14}$$

$$\sigma_{11} \equiv 24M_0^2\ln\frac{\Lambda^2}{M_0^2} - \frac{477}{2}\sigma_1^2 - 15\sqrt{3}\sigma_1\sigma_2 + \frac{9}{2}\sigma_2^2,$$
$$\sigma_{12} \equiv -\frac{15\sqrt{3}}{2}\sigma_1^2 + 9\sigma_1\sigma_2 + \frac{3\sqrt{3}}{2}\sigma_2^2,$$
$$\sigma_{22} \equiv \frac{9}{2}\sigma_1^2 + 3\sqrt{3}\sigma_1\sigma_2 + \frac{27}{2}\sigma_2^2.$$

Here $M_0 \equiv M(0) = 2\sigma_1$ is the dynamic quark mass at zero external momentum. The remaining logarithmic divergences will be absorbed later by meson masses and renormalization of meson fields.

The physical mass spectrum is defined by the secular equation

$$\det(\hat{A}p^2 + \hat{B})^{(\rho,a)} = 0, \qquad m^2_{\text{phys}} = -p_0^2. \tag{15}$$

As will be seen further on the consistency with CSR sum rules imposes the following scale condition:

$$\bar{\Delta}_{kl} = O\left(\Lambda^2\right). \tag{16}$$

Using (12), (13), one has for eq. (15)

$$6\left(\ln\frac{\Lambda^2}{M_0^2} - 2\right)p^4 - \left(8\ln\frac{\Lambda^2}{M_0^2}\bar{\Delta}_{22} - 15\bar{\Delta}_{22} + 3\bar{\Delta}_{11} + 2\sqrt{3}\bar{\Delta}_{12}\right)p^2 + 4\det\hat{\bar{\Delta}} = 0, \tag{17}$$

The solution of eq. (17) in the large-log approximation $\ln\frac{\Lambda^2}{M_0^2} \gg 1$ is as follows:

$$m_\rho^2 = -\frac{\det\hat{\bar{\Delta}}}{2\ln\frac{\Lambda^2}{M_0^2}\bar{\Delta}_{22}} + c_1 + O\left(\frac{M_0^2}{\ln\frac{\Lambda^2}{M_0^2}}\right), \tag{18}$$

$$m_{\rho'}^2 = -\frac{4}{3}\bar{\Delta}_{22} + \delta + c_2 + O\left(\frac{M_0^2}{\ln\frac{\Lambda^2}{M_0^2}}\right). \tag{19}$$

To obtain the A-meson mass spectrum, it is enough to make the replacement (see eqs. (13), (14)) $\bar{\Delta}_{kl} \to \bar{\Delta}_{kl} - 1/2\sigma_{kl}$. The result is

$$m_{a_1}^2 = -\frac{\det\hat{\bar{\Delta}}}{2\ln\frac{\Lambda^2}{M_0^2}\bar{\Delta}_{22}} + 6M_0^2 + c_1 + O\left(\frac{M_0^2}{\ln\frac{\Lambda^2}{M_0^2}}\right), \tag{20}$$

$$m_{a_1'}^2 = -\frac{4}{3}\bar{\Delta}_{22} + 3\bar{\sigma} + \delta + c_2 + O\left(\frac{M_0^2}{\ln\frac{\Lambda^2}{M_0^2}}\right). \tag{21}$$

The prime labels everywhere the corresponding excited meson state and we have introduced the notations

$$\delta \equiv -\frac{6m_\rho^2\ln\frac{\Lambda^2}{M_0^2} + d}{6\ln\frac{\Lambda^2}{M_0^2}}, \quad c_1 \sim c_2 = O\left(\frac{\Lambda^2}{\ln^2\frac{\Lambda^2}{M_0^2}}\right), \tag{22}$$

$$d \equiv 3\bar{\Delta}_{11} + 2\sqrt{3}\bar{\Delta}_{12} + \bar{\Delta}_{22},$$
$$\bar{\sigma} \equiv \sigma_1^2 + \frac{2\sqrt{3}}{3}\sigma_1\sigma_2 + 3\sigma_2^2 > 0. \tag{23}$$

As is seen from eqs. (18)-(21) the scale of mass squared for ground VA states is $O(\Lambda^2/\ln\frac{\Lambda^2}{M_0^2})$ and for excited ones is $O(\Lambda^2)$. Thus, the excited states turn out to be logarithmically heavier than the ground ones as was for the scalar (S) and pseudoscalar (P) case [16]. This qualitative property is independent of any concrete choice of form factors. Combining eqs. (18)-(21) with the corresponding results in [16,17], one obtains

$$m_{a_1}^2 - m_\rho^2 = 6M_0^2 + O\left(\frac{M_0^2}{\ln\frac{\Lambda^2}{M_0^2}}\right) = \frac{3}{2}(m_\sigma^2 - m_\pi^2) + O\left(\frac{M_0^2}{\ln\frac{\Lambda^2}{M_0^2}}\right), \tag{24}$$

$$m_{a_1'}^2 - m_{\rho'}^2 = 3\bar{\sigma} + O\left(\frac{M_0^2}{\ln\frac{\Lambda^2}{M_0^2}}\right) = \frac{3}{2}(m_{\sigma'}^2 - m_{\pi'}^2) + O\left(\frac{M_0^2}{\ln\frac{\Lambda^2}{M_0^2}}\right). \tag{25}$$

In the large-log approach the last equalities in eqs. (24), (25) do not depend on model parameters. We note also that differences of masses squared both in eq. (24) and in eq. (25) are of order $O(M_0^2)$, which indicates the chiral-symmetry restoration at a scale over

1 GeV. It can be shown also by corresponding fits. Having as input $\Lambda = 1000$ MeV and

$$M_0 = 2\sigma_1 = 320\,\mathrm{MeV}\,,$$
$$\langle\bar{q}q\rangle \simeq -\frac{N_c\Lambda^2}{8\pi^2}(\sigma_1 - \sqrt{3}\,\sigma_2) = -(250\,\mathrm{MeV})^3, \tag{26}$$

one can fix

$$\sigma_1 = 160\,\mathrm{MeV}\,, \qquad \sigma_2 = -145\,\mathrm{MeV}\,. \tag{27}$$

From eq. (25) one obtains the mass splittings

$$m_{\sigma'} - m_{\pi'} \approx 45\,\mathrm{MeV}\,, \qquad m_{a_1'} - m_{\rho'} \approx 60\,\mathrm{MeV}\,, \tag{28}$$

which show a fast restoration of the chiral symmetry. We confront the π'- and ρ'-mesons with the states [4] $\pi(1300)$ ($m_{\pi'} = 1300 \pm 100$ MeV) and $\rho(1450)$ ($m_{\rho'} = 1465 \pm 25$ MeV) correspondingly. Then the QQM predicts the mass of scalar state $m_{\sigma'}$ in the energy region 1250–1450 MeV and for the axial-vector state $m_{a_1'}$ in the energy range 1500–1550 MeV. The former particle can be identified with the scalar state $f_0(1500)$ [4–6,18,19]. Since the unitarization effects are not taken into account, one should compare the predictions with bare masses if they are estimated somehow. For example, the bare mass of $f_0(1500)$ is approximately 1230 MeV [6]. Moreover, the real scalar resonances have an admixture of $\bar{s}s$ components, which we do not consider here. The mass of the latter particle is not yet finally established.

Equation (24) at $m_\rho = 770$ MeV gives $m_{a_1} = 1100$ MeV, which is close to the prediction of the Weinberg relation $m_{a_1} = \sqrt{2}\,m_\rho$. The experimental data yield [4] $m_{a_1} = 1230 \pm 40$ MeV, that is, the model prediction is within the large-N_c approximation. On the other hand, eq. (24) predicts the mass of the lightest scalar meson to be about 650 MeV. This particle was often confronted with the broad scalar state [4] $f_0(600)$. However, from the modern point of view this broad state is regarded as a dynamic resonance [6,19]. The mass of the ground scalar meson in the model may be rather confronted with a bare mass of the scalar state $f_0(980)$ [6], without an admixture of $\bar{s}s$ quarks.

Let us comment the approximations used to derive the meson mass spectrum: namely, the large-N_c and leading-log approximations. The first one is equivalent [12] to the neglect of meson loops. The second one fits well the quarks confinement as quark-antiquark threshold contributions are suppressed in two-point functions in the leading-log approximation. The accuracy of this approximation is controlled also by the magnitudes of higher-dimensional operators neglected in the QQM, *i.e.* by the contributions of heavy-mass resonances not included in the QQM. All these approximations are mutually consistent. In particular, in the effective action without gluons the quark confinement should be realized with the help of an infinite number of quasilocal vertices with higher-order derivatives. Then the imaginary part of quark loops can be compensated and their momentum dependence can eventually reproduce the infinite sum of meson resonances in the large-N_c limit. If the effective action is truncated with a finite number of vertices and thereby deals with only a few resonances one has to retain only a finite number of terms in the low-momentum expansion of quark loops in the CSB phase, with a non-zero dynamic mass.

3 Chiral-symmetry restoration sum rules and constraints on parameters of QQM

In this section we exploit the constraints based on chiral-symmetry restoration in QCD at high energies for the VA $SU(2)$ QQM. As was mentioned in the introduction, at intermediate energies the correlators of the QQM can be matched to the OPE of QCD correlators. In the large-N_c limit the correlators of color-singlet quark currents are saturated by narrow meson resonances. In particular, the two-point correlators of vector and axial-vector quark currents are represented by the sum of related meson poles in Euclidean space:

$$\Pi^{\mathrm{C}}(p^2) = \int \mathrm{d}^4x \exp(ipx)\langle\bar{q}\Gamma q(x)\bar{q}\Gamma q(0)\rangle =$$
$$\sum_n \frac{Z_n^{\mathrm{C}}}{p^2 + m_{\mathrm{C},n}^2} + D_0^{\mathrm{C}} + D_1^{\mathrm{C}} p^2, \tag{29}$$
$$\mathrm{C} \equiv \mathrm{V,A}; \qquad \Gamma = \gamma_\mu, \gamma_\mu\gamma_5; \qquad D_0, D_1 = \mathrm{const}\,.$$

The last two terms represent a perturbative contribution, with D_0 and D_1 being contact terms required for the regularization of infinite sums. On the other hand, their high-energy asymptotics is provided [13] by the perturbation theory and the operator product expansion due to asymptotic freedom of QCD. Therefrom, the above correlators increase at large p^2,

$$\Pi^{\mathrm{C}}(p^2)\mid_{p^2\to\infty} \sim p^2 \ln\frac{p^2}{\mu^2}\,. \tag{30}$$

When comparing the two expressions above, one concludes that the infinite series of resonances with the same quantum numbers should exist in order to reproduce the perturbative asymptotics.

Meantime the differences of correlators for opposite-parity currents rapidly decrease at large momenta $p^2 \to \infty$ [13,20]

$$\Pi^{\mathrm{V}}(p^2) - \Pi^{\mathrm{A}}(p^2) \equiv \frac{\Delta_{\mathrm{VA}}}{p^6} - \frac{m_0^2\Delta_{\mathrm{VA}}}{p^8} + O\left(\frac{1}{p^{10}}\right),$$
$$\Delta_{\mathrm{VA}} \simeq -16\pi\alpha_s\langle\bar{q}q\rangle^2, \tag{31}$$

where $m_0^2 = 0.8 \pm 0.2\,\mathrm{GeV}^2$ [21] and we have defined in the V, A channels

$$\Pi_{\mu\nu}^{\mathrm{V,A}}(p^2) \equiv (-\delta_{\mu\nu}p^2 + p_\mu p_\nu)\Pi^{\mathrm{V,A}}(p^2). \tag{32}$$

Therefore, the chiral symmetry is restored at high energies and difference (31) represents an order parameter of chiral-symmetry breaking in QCD. As it decreases rapidly at large momenta, one can perform the matching of QCD asymptotics by means of few lowest-lying resonances. This procedure gives a number of constraints from the chiral-symmetry restoration. They may be used both for obtaining some additional bounds on the model parameters and for calculating some decay constants.

Expanding the meson correlators (29) in powers of p^2, one arrives at the CSR sum rules

$$\begin{aligned}
&\sum_n Z_n^{\mathrm{V}} - \sum_n Z_n^{\mathrm{A}} = 4F_\pi^2 \,,\\
&\sum_n Z_n^{\mathrm{V}} m_{\mathrm{V},n}^2 - \sum_n Z_n^{\mathrm{A}} m_{\mathrm{A},n}^2 = 0 \,,\\
&\sum_n Z_n^{\mathrm{V}} m_{\mathrm{V},n}^4 - \sum_n Z_n^{\mathrm{A}} m_{\mathrm{A},n}^4 = \Delta_{\mathrm{VA}} \,,\\
&\sum_n Z_n^{\mathrm{V}} m_{\mathrm{V},n}^6 - \sum_n Z_n^{\mathrm{A}} m_{\mathrm{A},n}^6 = -m_0^2 \Delta_{\mathrm{VA}} \,.
\end{aligned} \tag{33}$$

The first two relations are the Weinberg sum rules. The quantity F_π is the pion decay constant ($F_\pi \simeq 87$ MeV in the chiral limit) which is equal in the QQM, with the P-A mixing effect being taken into account [15]:

$$F_\pi^2 = \frac{N_f N_c M_0^2 m_\rho^2}{4\pi^2 m_{a_1}^2} \ln \frac{\Lambda^2}{M_0^2} + O(M_0^2). \tag{34}$$

For the model under consideration relation (34) fixes the logarithm of the cutoff in terms of physical parameters.

The residues in resonance pole contributions in the vector and axial-vector correlators have the structure,

$$Z_n^{(\mathrm{V,A})} = 4 f_{(\mathrm{V,A}),n}^2 m_{(\mathrm{V,A}),n}^2 \,, \tag{35}$$

with $f_{(\mathrm{V,A}),n}$ being defined as electromagnetic decay constants.

We note that for the two-channel case the system (33) can be solved explicitly. For example, for the inputs $\langle \bar{q}q \rangle = -(250\,\mathrm{MeV})^3$, $m_0 = 1000$ MeV, $m_\rho = 770$ MeV, $m_{a_1} = 1230$ MeV, $m_{\rho'} = 1460$ MeV, $m_{a_1'} = 1640$ MeV one obtains $f_\rho = 0.19$, $f_{a_1} = 0.14$, $f_{\rho'} = 0.11$, $f_{a_1'} = 0.06$. Also the constant f_{a_1} turns out to be slightly larger than its experimental value ($f_{a_1} = 0.10 \pm 0.02$) and the constants $f_{\rho'}$, $f_{a_1'}$ are not known yet, the solution looks reasonable. In particular, for the chiral constant L_{10} (see below) this solution gives $L_{10} = -6.25 \cdot 10^{-3}$.

In order to get some constraints on parameters of VA $SU(2)$ QQM, we calculate the corresponding two-point correlators by the variation of external sources for auxiliary boson fields. Let us show the appropriate scheme of calculations for the V case. Taking into account the external vector sources $V_{k,\mu}^i$, the Lagrangian reads as follows:

$$\begin{aligned}
\mathcal{L}_{\mathrm{aux}}^{\mathrm{V}} = \bar{q} \left(\not{D} + i \sum_{k=1}^{2} \Gamma_{\mathrm{V},k}^i V_{k,\mu}^i \right) q \\
+ N_f N_c \Lambda^2 \sum_{k,l=1}^{2} \rho_{k,\mu}^i b_{kl}^{-1} \rho_{l,\mu}^i \,.
\end{aligned}$$

After shifting the bosonic fields

$$\rho_{k,\mu}^i \longrightarrow \rho_{k,\mu}^i - V_{k,\mu}^i \,,$$

and integrating over fermionic degrees of freedom, one comes to the following effective action in external vector sources:

$$\begin{aligned}
S_{\mathrm{eff}}^{\mathrm{V}}(\rho_{k,\mu}^i, V_{k,\mu}^i) = -N_f N_c \mathrm{Tr} \ln \not{D} + N_f N_c \Lambda^2 \\
\times \int \mathrm{d}^4 x \sum_{k,l=1}^{2} b_{kl}^{-1} \left\{ \rho_{k,\mu}^i \rho_{l,\mu}^i - 2 V_{k,\mu}^i \rho_{l,\mu}^i + V_{k,\mu}^i V_{l,\mu}^i \right\} .
\end{aligned}$$

Expanding $\mathrm{Tr} \ln \not{D}$ up to quadratic in fields terms, one has

$$\begin{aligned}
S_{\mathrm{eff}}^{(2)}(\rho_{k,\mu}^i, V_{k,\mu}^i) = \sum_{k,l=1}^{2} \Bigg\{ \frac{1}{2} \rho_{k,\mu}^i C_{kl}^{(\rho)\mu\nu} \rho_{l,\nu}^i \\
+ N_c \Lambda^2 b_{kl}^{-1} \left[-2 V_{k,\mu}^i \rho_{l,\mu}^i + V_{k,\mu}^i V_{l,\mu}^i \right] \Bigg\},
\end{aligned}$$

where $C_{kl}^{(\rho)\mu\nu}$ is given by eq. (11). Introducing the vectors

$$\rho_\mu \equiv \begin{pmatrix} \rho_{1,\mu}^i \\ \rho_{2,\mu}^i \end{pmatrix} , \qquad V_\mu \equiv \begin{pmatrix} V_{1,\mu}^i \\ V_{2,\mu}^i \end{pmatrix}$$

and taking into account (10) (where we neglect the last term), one integrates over ρ_μ with the result

$$\begin{aligned}
\frac{12\pi^2}{N_f N_c} S_{\mathrm{eff}}(V_\mu) = -\frac{9}{8} \left(\Lambda^4 + O(\Lambda^2) \right) V_\mu^T \hat{H}_{\mu\nu}^\rho V_\nu \\
+ \frac{3\Lambda^2}{4} V_\mu^T V_\nu \delta_{\mu\nu} \,,
\end{aligned} \tag{36}$$

$$\begin{aligned}
\hat{H}_{\mu\nu}^\rho \equiv \left(\hat{A} p^2 + \hat{B}^\rho \right)^{-1} \hat{A} \left(\hat{B}^\rho \right)^{-1} \\
\times (-p^2 \delta_{\mu\nu} + p_\mu p_\nu) + \left(\hat{B}^\rho \right)^{-1} \delta_{\mu\nu} \,.
\end{aligned} \tag{37}$$

The last term in eq. (37) together with that in (36) form a local term which will be cancelled by the same term in the A case. Substituting the identity

$$\bar{q} \gamma_\mu q = \frac{1}{2} \left(\bar{q} f_1(s) \gamma_\mu q - \sqrt{3}\, \bar{q} f_2(s) \gamma_\mu q \right)$$

into the vector correlator

$$\Pi_{\mu\nu}^\rho(p^2) = 4 \int \mathrm{d}^4 x \exp(ipx) \langle \bar{q} \gamma_\mu q(x) \bar{q} \gamma_\nu q(0) \rangle, \tag{38}$$

the latter can be rewritten through the second variational derivatives:

$$\begin{aligned}
\Pi_{\mu\nu}^{(\rho)}(p^2) = \frac{N_f N_c}{12\pi^2} \left[\Pi_{11}^{(\rho)} + 3\Pi_{22}^{(\rho)} - 2\sqrt{3}\, \Pi_{12}^{(\rho)} \right] \\
\times (-\delta_{\mu\nu} p^2 + p_\mu p_\nu) \,,
\end{aligned} \tag{39}$$

$$\hat{\Pi}^{(\rho)} \equiv \left(\hat{A} p^2 + \hat{B}^\rho \right)^{-1} \hat{A} \left(\hat{B}^\rho \right)^{-1} . \tag{40}$$

On the other hand, this correlator is parametrized as follows (see eq. (29)):

$$\Pi_{\mu\nu}^{(\rho)}(p^2) = \left[\frac{Z_\rho}{p^2 + m_\rho^2} + \frac{Z_{\rho'}}{p^2 + m_{\rho'}^2} \right] (-\delta_{\mu\nu} p^2 + p_\mu p_\nu) \,. \tag{41}$$

The comparison of (39) and (41) allows to obtain the corresponding residues (see eqs. (45)). In the mean-field approximation the vector correlator and residues can be calculated exactly [22].

The A-mesons must be considered together with the P ones due to the P-A mixing. The relevant term in the effective action

$$S^{(2)}_{\rm eff}(\pi^i_k, a^i_{k,\mu}) = \frac{1}{2}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\sum_{k,l=1}^{2} 2\pi^i_k C^{\pi a,\mu}_{kl} a^i_{l,\mu}\,,$$

appears by virtue of the non-zero value of the corresponding second variation of $S_{\rm eff}(\pi^i_k, a^i_{k,\mu})$:

$$\begin{aligned} C^{\pi a,\mu}_{kl} &= -N_f N_c \int \frac{\mathrm{d}^4q}{(2\pi)^4}\,\mathrm{tr}\left\{(i\gamma_5)\frac{f_k\left(\left(\frac{q+p/2}{\Lambda}\right)^2\right)}{\not q+\frac{1}{2}\not p + iM}\right. \\ &\quad\left.\times(i\gamma_\mu\gamma_5)\frac{f_l\left(\left(\frac{q+p/2}{\Lambda}\right)^2\right)}{\not q-\frac{1}{2}\not p + iM}\right\} \\ &= -\frac{4iN_fN_c}{(2\pi)^4}\int\frac{\mathrm{d}^4q M f_k\left(\frac{q^2}{\Lambda^2}\right) f_l\left(\frac{q^2}{\Lambda^2}\right)}{\left[\left(q+\frac{1}{2}p\right)^2+M^2\right]\left[\left(q-\frac{1}{2}p\right)^2+M^2\right]}p_\mu \\ &\quad + O\left(\frac{p^3M_0}{\Lambda^2}\right). \end{aligned} \tag{42}$$

In order to exclude the mixing terms, one makes the shift of the A fields:

$$a^i_{k,\mu} \longrightarrow a^i_{k,\mu} + D^i_{kl}\pi^i_l p_\mu\,,$$

with the elements D^i_{kl} being defined by the requirement of cancellation of P-A terms. This leads to changing the kinetic matrix $\hat{A}^\pi$ due to contribution of the longitudinal A part:

$$\begin{aligned}\hat{A}^\pi &\longrightarrow \hat{A}^\pi_{\rm ren} = \\ &\times\begin{pmatrix} 4\frac{m_\rho^2}{m_{a_1}^2}\ln\frac{\Lambda^2}{M_0^2}+O(1) & -\frac{\sqrt{3}}{2}+O\left(\frac{M_0^2\ln\frac{\Lambda^2}{M_0^2}}{\Lambda^2}\right) \\ -\frac{\sqrt{3}}{2}+O\left(\frac{M_0^2\ln\frac{\Lambda^2}{M_0^2}}{\Lambda^2}\right) & \frac{3}{2}+O\left(\frac{M_0^2}{\Lambda^2}\right)\end{pmatrix},\end{aligned} \tag{43}$$

where eq. (24) and the scale (16) have been exploited. It is easy to check that the redefinition (43) does not influence the mass spectrum of the P-mesons.

As a result, one finds the residues in the meson poles for VA boson states:

$$\begin{aligned} 4F_\pi^2 &\simeq \frac{(m_{a_1}^2-m_\rho^2)\,\delta}{m_\rho^2 m_{a_1}^2 m_{a_1'}^2}\,Z_1\,, \qquad Z_1 \equiv \frac{3N_cN_f\Lambda^4}{16\pi^2}\,, \\ Z_\rho &\simeq 4F_\pi^2\frac{m_{a_1}^2}{m_{a_1}^2-m_\rho^2}\,, \end{aligned} \tag{44}$$

$$Z_{a_1} \simeq 4F_\pi^2\frac{m_\rho^2}{m_{a_1}^2-m_\rho^2}\,,\quad Z_{\rho'} \simeq \frac{Z_1}{m_{\rho'}^2}\,,\quad Z_{a_1'} \simeq \frac{Z_1}{m_{a_1'}^2}\,, \tag{45}$$

where δ is given by (22). In contrast to the situation in the SP case, the residues in the VA poles are of the same order of magnitude:

$$Z_\rho \sim Z_{a_1} \sim Z_{\rho'} \sim Z_{a_1'} = O\left(\Lambda^2\right).$$

The statement (16) follows from the comparison of eqs. (34), (44).

The first and the second sum rules are satisfied identically. The third one takes the form

$$Z_1\left(m_{a_1'}^2 - m_{\rho'}^2\right) \simeq 16\pi\alpha_s\langle\bar{q}q\rangle^2 \qquad \text{or} \qquad 3Z_1\bar{\sigma} = -\Delta_{\rm VA}\,. \tag{46}$$

This relation represents the constraint on the vector QQM parameters following from the OPE. We note that the analog of eq. (46) in the scalar case [16] may be cast into the form

$$\begin{aligned}\frac{N_fN_c\Lambda^4}{2\pi^2}\left(m_{\sigma'}^2 - m_{\pi'}^2\right) &\simeq 24\pi\alpha_s\langle\bar{q}q\rangle^2 \qquad \text{or} \\ 3Z_1\bar{\sigma} &= -\frac{27}{32}\Delta_{\rm VA}\,.\end{aligned} \tag{47}$$

The minor discrepancy between relations (46) and (47) is about 16% and can be referred to the quality of the four-resonance approximation. Combining eqs. (46), (47) one can obtain two independent estimations of the quantity:

$$\frac{m_{a_1'}^2 - m_{\rho'}^2}{m_{\sigma'}^2 - m_{\pi'}^2} \approx \begin{cases} 1.5 & \text{from QQM}\,, \\ 1.8 & \text{from CSR}\,. \end{cases} \tag{48}$$

This shows that the saturation of two-point correlators by four resonances is quite robust.

The fourth sum rule looks as follows in the large-log approach [18]:

$$m_{a_1'}^2 \simeq m_{\rho'}^2 \simeq \frac{m_0^2}{2} \quad \text{or} \quad -\frac{4}{3}\bar{\Delta}_{22}\cdot 3\bar{\sigma} \simeq -\frac{m_0^2\Delta_{\rm VA}}{2Z_1}\,. \tag{49}$$

Numerical estimations [18] show that the last sum rule fails for the QQM with the ground and first-excited sets of VA-mesons.

The structure of $Z_{\rho'}$ and $Z_{a_1'}$ shows that if $m_{a_1'} \simeq m_{\rho'}$ then $Z_{a_1'} \simeq Z_{\rho'}$ and, therefore, $f_{a_1'} \simeq f_{\rho'}$. Thus, these residues approximately cancel each other in the sum rules (in the large-log approach) and one arrives at the one-channel results for f_ρ and f_{a_1} [15] (see also [7]),

$$f_\rho \simeq \frac{F_\pi m_{a_1}}{m_\rho\sqrt{m_{a_1}^2-m_\rho^2}}\,, \qquad f_{a_1} \simeq \frac{F_\pi m_\rho}{m_{a_1}\sqrt{m_{a_1}^2-m_\rho^2}}\,. \tag{50}$$

After evaluating, we get $f_\rho \approx 0.15$ and $f_{a_1} \approx 0.06$, to be compared with the experimental value [23] from the decay $\rho^0 \to e^+e^-$, $f_\rho = 0.20\pm0.01$, and from the decay $a_1 \to \pi\gamma$, $f_{a_1} = 0.10\pm0.02$. We can also calculate the chiral constant L_{10} which appears in the effective chiral Lagrangian [24] and which is related to the mean-square electromagnetic pion radius and to the axial-vector pion form factor $F_{\rm A}$

for the decay $\pi \to e\nu\gamma$ (see, for example, [25]). Namely, for the ground VA states one gets

$$L_{10} = \frac{1}{4}\left(\sum_n f_{\mathrm{A},n}^2 - \sum_n f_{\mathrm{V},n}^2\right) \simeq \frac{1}{4}\left(f_{a_1}^2 - f_\rho^2\right) \approx -4.7 \cdot 10^{-3},$$

to be compared with the result of [3] from hadronic τ-decays: $L_{10} = -(6.36 \pm 0.09|_{\mathrm{expt}} \pm 0.16|_{\mathrm{theor}}) \cdot 10^{-3}$.

It is worth mentioning that using the first three CSR sum rules (33) and the requirement of a fast chiral-symmetry restoration, one obtains the estimate [7] $L_{10} \approx -6.0 \cdot 10^{-3}$ and, for the electromagnetic pion-mass difference, $\Delta m_\pi^{(4)}|_{\mathrm{em}} \approx 3.85 \pm 0.16$ MeV, which improves the agreement between theoretical predictions and the experimental value of $\Delta m_\pi|_{\mathrm{em}}^{\mathrm{expt}} \approx 4.42 \pm 0.03$ MeV (with a contribution due to the isospin symmetry breaking being taken into account, for details see [7]). Given a value for L_{10}, one may calculate the pion polarizability α_E (see, for example, [2]). For instance, for $L_{10} \approx -6.0 \cdot 10^{-3}$ one gets $\alpha_E = -2.72 \cdot 10^{-4}$ fm^3, to be compared with the result of [2] $\alpha_E = (-2.71 \pm 0.88) \cdot 10^{-4}$ fm^3. Thus, allowance for the contribution of higher meson resonances can be of importance in calculating some physical constants.

4 Summary and conclusions

We have shown that the $SU(2)$ Quasilocal Quark Model with chirally invariant four-fermion vector and axial-vector vertices including derivatives of fields can serve to describe the physics of vector and axial-vector meson resonances and their excitations at intermediate energies. The corresponding mass spectrum for the ground and first-excited vector and axial-vector boson states was derived in the mean-field, large-log approximations and in the chiral limit. The qualitative features of the mass spectrum obtained turn out to be the same as in the scalar-pseudoscalar case [16]: the excited states are logarithmically heavier than the ground ones and a fast restoration of the chiral symmetry over the scale 1 GeV is predicted (this fact was confirmed also by our numerical estimates: $m_{\sigma'} - m_{\pi'} \simeq 45$ MeV and $m_{a_1'} - m_{\rho'} \simeq 60$ MeV). The comparison with the $SU(2)$ scalar-pseudoscalar QQM has allowed to obtain two remarkable relations between boson masses independent of the model parameters (see eqs. (24), (25)):

$$m_{a_1}^2 - m_\rho^2 \simeq \frac{3}{2}(m_\sigma^2 - m_\pi^2),$$

$$m_{a_1'}^2 - m_{\rho'}^2 \simeq \frac{3}{2}(m_{\sigma'}^2 - m_{\pi'}^2).$$

We obtained the following estimates: $m_{a_1'} = 1500$–1550 MeV (which can be identified with the $a_1(1640)$-meson [4]) and $m_{\sigma'} = 1250$–1450 MeV (which can be identified with the bare mass of the $f_0(1500)$-meson [6], without an admixture of $\bar{s}s$ components).

To realize the correspondence (matching) with QCD at intermediate energies, where the OPE can be applied, the chiral-symmetry restoration sum rules were imposed on the vector–axial-vector $SU(2)$ QQM. For the four-resonance ansatz some constraints on model parameters were obtained. The residues for the ground and excited vector–axial-vector states turn out to be of the same order of magnitude, as opposed to the SP case. The inclusion of excited states does not change significantly the electromagnetic decay constants of the ground vector–axial-vector states as compared with the one-channel results: $f_\rho \simeq 0.15$ and $f_{a_1} \simeq 0.06$ (see, however, [7]).

The $U(3)$ extension of the scalar-pseudoscalar-vector-axial-vector QQM is in progress and the preliminary results can be found in [17,18]. In the vector–axial-vector sector the agreement with the experimental data [4] turns out to be within 10% for the vector mesons and 15% for the axial-vector ones.

Thus, the Quasilocal Quark Model describes reasonably well the spectral characteristics for vector and axial-vector mesons and their first excitations and fits the phenomenology of low-energy meson physics. The matching to non-perturbative QCD based on the chiral-symmetry restoration at high energies (CSR sum rules) improves the predictability of the QQM and leads to some constraints on its parameters.

Finally, we would like to mention other possible applications of the QQM. First, such models are thought of as relevant for the investigations of the behaviour of hadron matter at high temperatures and nuclear densities in the region near the restoration of chiral symmetry. One could expect that the mass-splittings shrink in response to an increase in the quark density value and, therefore, the excitations become lighter and more important in hadron kinetics. Second, the QQM can be used to describe the Higgs particles in extensions of the Standard Model [26,27].

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